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An Introduction to Artificial Neural Networks in Ordinary Differential Equations: A Harrod-Domar Growth-Based Approach

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Abstract:

In many scientific and engineering fields, ordinary differential equations (ODEs) are essential for modeling a broad variety of phenomena. Even if they work well, traditional numerical techniques for solving ODEs can be computationally demanding and may not work well for complex or high-dimensional situations. This study highlights the crucial role that ODEs play in influencing modern developments by creating a link between theoretical underpinnings and practical applications. ODEs have revolutionized deep neural network training methodologies. Even if they work well, traditional numerical techniques for solving ODEs can be computationally demanding and may not work well for complex or high-dimensional situations. Because of its capacity to imitate intricate patterns and functions, Artificial Neural Networks (ANNs) have recently surfaced as a viable substitute for solving ODEs. They make it possible for dynamic topologies, in which the behavior of the network changes continually over time, enhancing its capacity to recognize intricate patterns and adjust to shifting data. The mathematics behind these applications is examined in this essay, with a focus on ODE critical contributions to the development of artificial intelligence as well as their importance in influencing the current scientific and economic environment. This article's goal is to give scholars and students interested in this significant subject a clear and succinct guide.

Keywords: ODEs, derivatives, differential equations, Constant coefficients,

1. Introduction

Numerous differential equations that were difficult to solve analytically prompted research into numerical approximation techniques. The requirement to perform the calculations by hand or with somewhat antiquated computing equipment significantly limited the implementation of numerical integration techniques, which by 1900 had been developed in a pretty effective manner. The spectrum of problems that can be successfully investigated by numerical methods has greatly expanded after World War II due to the advent of ever-more-powerful and adaptable computers. During the same time period, extremely sophisticated and reliable numerical integrators were created, and they are currently widely accessible, even on smartphones and other mobile devices. These technological advancements have made it possible for individual students to solve a wide range of important challenges.

In a variety of fields, such as physics, engineering, biology, and finance, ordinary differential equations, or ODEs, are essential to mathematical modeling. Whether anticipating the motion of physical objects, the dynamics of biological processes, or the volatility of financial markets, these equations are crucial for understanding how a system changes over time. The interest in ODEs dates reverse to 17th century, when well-known mathematicians similar to Newton and Leibniz started investigating how they may be used to explain motion and change once calculus was developed. ODEs are elegant and versatile because they can show how quickly the state variables of a system change in relation to an independent variable, typically time. They are therefore a potent prediction tool in a variety of scientific fields, enabling us to forecast and understand how systems change over time. Accurate and effective ODE solving is essential for comprehending and predicting the behavior of such systems.

The foundation for solving ODEs has been traditional numerical techniques. To get fairly accurate solutions, methods including Runge-Kutta and finite distinction methods have been widely used. While the Runge-Kutta methods, particularly the well-liked fourth order variant, offer improved accuracy by taking into account many points within each step, Euler's method provides an easy way by approximating solutions through a sequence of discrete steps. Differential equations are frequently solved on grids and discredited using finite difference techniques. Despite being widely used and well-established, these approaches can face major difficulties when dealing with stiff equations, complex boundary conditions, or high-dimensional situations.

Increased computing expenses and difficulties in maintaining numerical stability might result from high-dimensional situations. Because of their quickly changing solutions, stiff equations frequently call for specific methods to handle their inherent numerical challenges. Understanding natural occurrences and creating predictive models is one of the main problems that ODEs attempt to solve. ODEs have a wide range of profound applications in the natural sciences, from fluid dynamics to celestial mechanics. ODEs, for example, can be used to model planetary orbits, pendulum motion, and epidemic spread, allowing physicists, epidemiologists, and astronomers to make precise predictions and decipher the complex mechanisms of the cosmos. Furthermore, because ODEs are widely used to build and optimize a wide range of technical breakthroughs, the large field of engineering owes a great deal to their ideas. ODEs are crucial to the safety, effectiveness, and operation of complex systems in mechanical systems, electrical circuits, and control theory. Among many other uses, the use of ODEs in engineering aids in the design of bridge that know how to support dynamic lots, the modeling of exciting circuit behavior in domestic appliance, and improvement of flight organize system stability in airplanes.

Ordinary differential equations are useful tools for assessing economic trends and creating financial models in the fields of economics and finance. ODEs have become a fundamental mathematical framework that supports important financial decisions, ranging as of the dynamics of supply market price to assessment of economic expansion. ODEs' usefulness is not limited to conventional scientific fields, though. They have an impact on a wide range of disciplines, including population dynamics, ecology, neurology, and even the study of human behavior and social relationships. ODEs give us a methodical way to comprehend complicated systems that behave dynamically by capturing the rate of modify connecting variables.

The study of ODEs is inextricably related to the larger field of mathematical analysis, despite their wide range of applications. ODE analytical solutions are prime examples of mathematical elegance that provide profound understanding of the complex relationships between mathematical structures and the real world. Furthermore, computational and numerical approaches become more prominent when analytical solutions are difficult to get, allowing us to investigate how ODEs behave in situations when precise solutions are not possible. This article's coverage of both the mathematical and physical interpretations and applications of these equations is one of its strong points. We go over how the solutions relate to physical systems and how they can be used to analyze and forecast how such systems will behave.

To demonstrate the uses of differential equations with constant coefficients throughout the equation, we offer examples from the fields of physics and engineering. This investigation seeks to understand the beauty and importance of ODEs in understanding dynamic events, whether they are microscopic or regulate largest scales in the universe. This study may squeeze intrinsic attractiveness of ODEs and lasting influence on the fields of science, engineering, and beyond by revealing the interaction between mathematical theory and real-world applications. Numerous academic disciplines, such as mathematics, physics, and engineering, rely heavily on the strategies and tactics discussed in this article.

The article may serve as a reference for resolving issues in related sectors and attempts to offer a thorough guide for scholars and students interested in this subject. Our goal in this essay is to present a thorough and in-depth analysis of the approaches taken to solve this class of equations. Our first step is to present the Standardized format of the constant coefficients and discuss their significance in resolving these equations. The characteristic equation and its roots, which reveal details about the nature of the solutions, are next presented. We go over the three scenarios: repeating roots, complex conjugate roots, and real and separate roots. We determine the general solution for every scenario and offer examples to show how the strategy is used.

2. Objectives

- To find the ordinary differential equations: A Model-Based Approach.
- To find the Harrod-Domar Growth model through a mathematical lens using ODEs.

3. Review of literature

Nurujjaman (2020) The primary attention is limited to interpreting these procedures geometrically and figuring out how to lower the inaccuracies. Ultimately, these techniques have been modified to increase performance by lowering errors, and the resulting approach has been called the Enhanced Euler's method. The work has discussed graphical results and numerical trials. Both graphical and numerical computations have been performed using MATLAB applications.

For imperfect temporal data modeling, Chang et al. (2023) present a unique continuous neural network architecture called temporal-aware Neural-Ordinary Differential Equations (TN-ODE). The suggested approach permits multi-step prediction at specified time points in addition to supporting imputation of missing values at arbitrary time points. Furthermore, a fully linked network is used to parameterize the derivative of latent states, allowing for the production of latent dynamics in continuous time. Data interpolation, extrapolation, and classification tasks are used to assess the suggested TN-ODE model on together synthetic and real-world partial time-series datasets.

Nair and associates (2025) over the last ten years, the field of neural differential equations has grown significantly, as seen by the rise in published literature. The capacity to simulate intricate, nonlinear systems and the possibility of extrapolation beyond the observable data are just two benefits of this methodology. This paper discusses neural network topologies used in NDEs, training approaches, and applications in a variety of fields. The current work provides instructions for mathematicians, computer scientists, and engineers by reviewing and classifying previous studies. It also covers some of the current issues and research topics in the subject of NDEs, as well as the benefits of NDEs.

Boyce and associates (2017) providing students with practical experience in the derivation of differential equations and persuading them that differential equations naturally occur in a wide range of real-world applications are the main goals of these applied problems. The book's repeated emphasis on the transportability of mathematical knowledge is another crucial idea. A particular solution method can be applied to any situation where that specific form of differential equation occurs, even though it is only applicable to a specific class of differential equations. We feel that it is no longer necessary to give particular examples of each sort of equation or solution method that we examine once this point has been persuasively conveyed. This choice enables us to maintain the main focus on the creation of more solution techniques for different kinds of differential equations while also keeping the book within a manageable size. Enhancing the readability and clarity of our introduction to differential equations and their applications is the aim. Furthermore, many captions have been added to better explain the figure's function without having to hunt through the surrounding text.

Coddington, (2012) This short book provides a comprehensive and methodical introduction to basic differential equations for science and math majors. The author extends the theory sufficiently throughout the book to provide assertions and evidence for the simpler existence and uniqueness theorems. Having taught at MIT, Princeton, and UCLA, Dr. Coddington has incorporated numerous activities aimed at improving students' equation-solving skills. In order to improve comprehension of the subject's mathematical structure and to introduce a number of pertinent subjects that are not addressed in the text, such as stability, equations with periodic coefficients, and boundary value issues, he has additionally provided questions (with answers).

Hamed and associates (2017) the accuracy of the numerical approach in resolving first-order ordinary differential equations was the focus of the study. Different step sizes and times (t) have been used to execute the Euler and Modified Euler procedures, and the local truncation errors have been computed. When we raised the time t, it was discovered that the numerical solutions were unstable and not quite analytical, and that the local truncation errors were excessive.

Ma and associates (2021) In contrast to continuous adjoint sensitivity analysis, we examine the performance features of Discrete Local Sensitivity Analysis implemented via Automatic Differentiation (DSAAD) in this work. While numerically uneven back solve techniques on or after the engine learning copy are shown to be inappropriate for the majority of scientific models. Furthermore, these results show that DSAAD is easily implemented and may be applied in unconventional ways to differential-algebraic equations, and hybrid DE structure where the instance and belongings of events depend on model restriction.

Lozada and associates (2021) The three goals of this paper's review of the literature on the teaching and learning of ordinary differential equations are to provide an overview of the field's current literature, aid in the integration of existing knowledge, and identify potential obstacles and viewpoints for future research on the subject. We used a methodology that combined bibliometric analysis with a thorough literature review. The following summarizes the paper's contributions: highlight the most recent findings in this field, describe the current research directions in the teaching and learning of ordinary differential equations, outline some issues that will be covered in the upcoming years, and establish a starting point for scholars who wish to pursue this line of inquiry.

Chen and associates (2025) In contrast to neural ordinary differential equations, a framework for neural ordinary differential equations based on incremental learning is presented in this research. This framework can improve learning capacity and establish the lowest data size needed for data modeling. This framework avoids the need for additional parameters by continuously updating the neural ODEs by newly acquired statistics. The smallest amount of data required for training can be ascertained once the predetermined accuracy has been attained. Additionally, the smallest amount of data needed for five traditional models at different sampling rates is examined.

Semnani and He (2024) in this study, we demonstrate the durability of neural differential equations in simulating elastoplastic path-dependent material behavior by using these freshly developed neural differential equations to model time series continuously. For generic time-variant dynamical systems, including path-dependent constitutive models, we create a novel sequential model known as INCDE. The analysis of INCDE is done in terms of convergence and stability. To illustrate the resilience, consistency, and correctness of the suggested method by finite element process with different monotonic loading procedures.

By combining denoising techniques to smooth the signal, sparse regression to find the pertinent parameters, and bootstrap confidence intervals to measure the uncertainty of the estimates, Egan et al. (2024) offer a way to find dynamical laws. We test our approach on well-known ordinary differential equations with variable signal-to-noise ratios, time series of increasing length, and an ensemble of random initial conditions. Our approach has the potential to influence the understanding of complex systems by automatically identifying dynamical systems with high accuracy, particularly in domains where data is plentiful but creating mathematical models requires a significant amount of work.

4. Data Analysis and Discussions

The highest derivative found in an equation is referred to as the order of an ODE. Stated differently, it shows the number of times the dependent variable of the function is differentiated. For example, just the first derivative is involved in a first-order ODE, the second derivative is involved in a second-order ODE, and so on. If an ODE can exist converted into a linear combination of dependent variable and its derivative, by coefficients that might be dependent on independent variable, then it is said to be linear. A linear ODE can be expressed mathematically as follows:

$$b_n(y)\frac{d^n x}{dy^n} + b_{n-1}(y)\frac{d^{n-1}x}{dy^{n-1}} + \cdots + b_1(y)\frac{dx}{dy} + b_0(y)x = g(y) \ldots .(1)$$

In this case, the independent variable is x, the forcing function is g(y), the coefficient functions are $b_i(y)$, and the unknown function is x. ODE analysis is made easier by linearity, which also permits solution superposition. The following is the typical form of n ODEs:

$$\frac{dx_1}{dz} = f_1 = (z, \mathrm{x}, x_2, \ldots x_n) \ldots .(2)$$

$$\frac{dx_2}{dz} = f_2 = (z, x_1, x_2, \ldots x_n) \ldots (3)$$

$$\frac{dx_n}{dz} = f_n = (z, x_1, x_2, \ldots x_n) \ldots ..(4)$$

in which by b $< z < a$ defines t. Equation 1's first solution is provided by:

$$\frac{dx}{dz} = f(z, x), x(0) = x_0 \ldots ..(5)$$

as well as

$$y = [x_1, x_2, \ldots x_n]^Z \ldots \ldots ..(6)$$

Since the unknown has a matrix dimension of n^*1 ,

$$f(z,x) = \begin{bmatrix} f_1 = (z, x_1, x_2, \ldots x_n) \\ f_2 = (z, x_1, x_2, \ldots x_n) \\ \vdots \\ f_n = (z, x_1, x_2, \ldots x_n) \end{bmatrix} \ldots \ldots (7)$$

A vector-valued function with an $o \times 1$ dimension is provided. Solutions that have been demonstrated to be both unique and existing are included in its initial value problem.

4.1 Introduction to Artificial Neural Networks (ANN)

The emergence of machine learning and artificial intelligence (AI) in recent years has brought new approaches to overcome the drawbacks of conventional approaches. An effective method for resolving ODEs among these is ANNs. ANNs are through up of layers of interrelated nodes, or neurons that are modeled after the neural architecture of the human brain. These networks have the ability to extract intricate patterns from data and use that knowledge to guide predictions. ANNs are a viable alternative for solving differential equations because of their inherent flexibility and adaptability, especially in situations where traditional methods are impractical or become onerous.

Ordinary Differential Equations (ODEs) can be solved by approximating the solution function using Artificial Neural Networks (ANNs). This method makes use of ANNs' capacity to extract intricate correlations and patterns from data. The ANN can efficiently learn the underlying ODE and produce an approximation solution by training the network to minimize the difference between the anticipated and actual solutions at different points in time.

In order to include ANNs into the process of solving ODEs, the network must be trained to either directly approximate the answer or discover the underlying patterns in data produced using numerical techniques. The ANN learns to predict solutions with a high degree of accuracy by supplying the network with inputs pertaining to the differential equation and desired outputs. Compared to traditional numerical approaches, this methodology may handle complicated or high-dimensional problems more effectively and provide a number of benefits, including lower processing costs.

In recent years, more scholars have turned to ANN to solve differential equations.

ANNs have the advantage of producing findings that can be differentiated from one another. It can also assist researchers in solving complex differential equations by avoiding iteration. To come up to the linear solution, a feed forward neural network be second-hand. Direct in addition to non-iterative feed forward neural network construction be bent using the hard bound commencement function. Problems with initial values and boundary values were solved by a trial approach that adhered to the specified requirements. The outcomes were contrasted with the finite element methodology, one of the most widely used numerical techniques.

The authors were able to arrive at accurate and ODE in a congested analytical formulation. The list of uses for ANNs was extended to include the working out of essential equations. In a work, Asady, and Nazarlue demonstrated how to effectively employ ANNs to estimate explanation in favor of linear two-dimensional Fredholm integral equations of 2nd class. ODEs are powerful tools for modeling and evaluating dynamic economic systems in the field of economics. Because ODEs capture the interaction of variables and their rates of modify, they supply important insights addicted to financial growth, policy impact, and market actions. One of the primary applications of ODEs is in economic growth models. This second order ODE looks at changes in production and capital stock as functions of instance to determine the symmetry status where these variables steadiness to enable augmentation. Solow-Swan model suggest a framework for understanding how economies change over instance and how various factors impact their pathways by incorporating variables like savings, depreciation, and technological advancement. ODEs are particularly crucial in two areas that are closely linked to economic research: population dynamics and demographic modeling. An understanding of these models is necessary to comprehend demographic shifts and how they affect the economy, including labor force trends and dependency ratios. Furthermore, by assisting in the development of models for economic aggregates like inflation, investment, and consumption, ODEs are essential to macroeconomic analysis. To examine how these two variables dynamically alter over time, ODEs can be used to simulate Phillips arc, which illustrates the connection among redundancy and price rises. This ODE-based method assists policymakers in manufacture educated choices on economic and financial policy in order to realize constant economic circumstances. Economic dynamics includes business cycle analysis as a subfield. Thanks to ODEs, economists may develop models that show how economic activity varies between expansionary and recessionary periods. By accounting for factors like government spending, investment, and consumption, these models mimic the cyclical patterns present in real economies. We can gain a better understanding of the underlying causes of economic oscillations and potential remedies by looking at these ODE-based models.

In conclusion, ordinary differential equations have a wide range of applications in economics. From macroeconomic trends and financial market dynamics to demographic studies and economic growth models, ODEs offer economists by a flexible toolkit to explore, elucidate, and foresee the complex relationships inside economic structure. By harnessing the power of geometric modeling, ODEs appreciably improve our comprehension of trade and industry events and guide the formulation of sustainable advance policies.

4.2 Harrod-Domar Growth Model

Ordinary differential equations (ODEs) provide a more quantitative representation of the Harrod-Domar growth model, which is undoubtedly a characteristic economic model. The relationship between investment, economic growth, and economic stability is examined by this model. The Harrod-Domar growth model can be described more mathematically as follows: Examine an economy that has the following characteristics: The GDP (gross domestic product) at time t is denoted by the formula $\mathrm{X(z)}$. The investment expenditure at time t is denoted by $\mathrm{I(z)}$. The customer's expenditure at time t is abbreviated as $\mathrm{C(z)}$. The capital stock data at time t is represented by $\mathrm{K(z)}$. Several important assumptions are made by the Harrod-Domar growth model:

- Changes in capital stock lead to investment:

$$\mathrm{I(z)} = \alpha \frac{dK}{dz} \ldots..(8)$$

where α is the capital output ratio, which is the amount of money needed to raise output by one unit.

- Spending on consumption is a percentage of GDP: In the equation

$$C(z) = cX \ldots..(9)$$

t, c represents the margins' consumption propensity.

- The difference between investment and depreciation is equal to the rate of modify of assets stock:

$$\frac{dK}{dz} = I(z) - \delta K(z) \ldots..(10)$$

where δ is rate of depreciation.

We may create a differential equation that depicts the economy's movement using these presumptions:

$$\frac{dX}{dz} = I(z) - C(z) = \frac{dK}{dz} - cX(z) \ldots.(11)$$

The following results from changing the equation for $I(z)$ and $C(z)$:

$$\frac{dX}{dz} = \alpha \frac{d^2K}{dz^2} - cX(z) \ldots.(12)$$

This second-ODE equation shows how rate of modify in GDP is related to both the existing GDP stage and velocity of modify in capital stocks. This formula encapsulates how investment and consumption interact to generate economic expansion. Initial conditions must be met in order to solve this ODE. For example, you may set the initial capital stock $K(0)$ and the initial GDP $X(0)$. When the ODE is solved, the original conditions, capital-output proportion α, and marginal inclination to put away c are used to mathematically describe how the economy's GDP develops over time. You may examine the economy's performance and stability over time with this model. This economy may grow steadily, oscillate, or be unstable depending on the values of α and c. The mathematical character of the model makes it possible to quantitatively analyze how many factors impact economic dynamics and stability. You can illustrate the Harrod-Domar growth model's use in applying ODEs to systematically and quantitatively examine the relationship between investment, consumption, and economic growth by exploring it from a more mathematical standpoint.

4.3 Solow Swan Model

By taking into account capital accumulation and technological advancement, the Solow-Swan growth model seeks to assess a nation's long-term economic growth. ODEs can be used to mathematically express the model as follows:

Describe the factors listed below: K(z) is the capital stock at time z. X(z) is the GDP at time z. L stands for those workers, who are taken to be constant. The consumption at time t is denoted by C(z). The savings rate, or the percentage of production saved, is denoted by s, and S(z) shows the decrease in savings as of time z. B is the rate of technical advancement, and B(z) is the scientific level at instance z. Among fundamental presumptions of Solow-Swan growth model are:

(i) Capital and labor are used to produce output with steady returns to scale. In mathematics,

$$X(z) = B(z) \cdot K(z)^{\alpha} \cdot L^{1-\alpha} \ldots.(13)$$

where α represents the production capital share.

(ii) $S(z) = s \cdot X(z)$ is the constant fraction s of output that is set aside for investment.

(iii) The growth rate of technological level B(z) is constant,

$$\text{b: } \frac{dA}{dt} = aB \ldots..(14)$$

These presumptions allow us to construct an ODE system that characterizes economic dynamics:

(i) The rate of modify of assets stock:

$$\frac{dK}{dz} = S(z) - \delta K(z) \ldots\ldots(15)$$

somewhere δ is reduction rate of assets.

(ii) The velocity of modify of yield:

$$\frac{dX}{dz} = \alpha B(z) K(z)^{\alpha-1} L^{1-\alpha} \frac{dK}{dz} \ldots..(16)$$

(iii) The rate of modify of using up:

$$\frac{dC}{dz} = (1-s)X(z) = (1-s)B(z)K(z)^{\alpha}L^{1-\alpha} \ldots\ldots(17)$$

(iv) The rate of modify of scientific level:

$$\frac{dB}{dz} = aB(z) \ldots.(18)$$

The relationships between the economy's output, consumption, technological advancement, and capital accumulation are all captured by this system of ODEs. You can examine the economy's steady-state capital level, long-term growth trajectory, and consumption dynamics under various parameter values by solving this system of equations. Analyzing the symmetry points where the charge of modify drop to zero, signifying a stable state—will help you examine the model's stability. These equilibrium points shed light on the economy's long-term trends, such as whether it oscillates or converges on a path of balanced growth. By using ODEs to approach the Solow-Swan growth model mathematically, you know how to demonstrate how these equations assist economists in quantitatively analyzing intricate relationships flanked by capital, expertise, and monetary growth, so illuminating the forces that shape a nation's expansion over instance

5. Conclusion

To sum up, field of ODEs offers an engrossing voyage through the complex fabric of dynamic systems and their uses. ODEs have become essential tools that connect theory and reality in a variety of fields, from deep neural networks to economic models. ODEs offer a rigorous framework for forecasting, comprehending change across time, and deciphering the intricate interactions between variables in a variety of domains. ODE-based designs in the field of deep neural networks provide new opportunities for modeling continuous transformations, boosting machine learning algorithms' ability to more accurately and effectively capture real-world occurrences. Economic models such as the Harrod-Domar ODEs demonstrate how they function as a mathematical compass that leads economists across the terrains of equilibrium, growth, and stability while providing insights into the underlying processes that form economies. The techniques for solving these equations, such as the characteristic equation and its roots, as well as the three potential scenarios of distinct and real roots, complicated conjugate roots, and repeated roots, have been presented in this article. In order to analyze and forecast the behavior of physical systems, we have additionally emphasized the solutions' physical interpretations and applications. For scholars and students interested in this subject, this article is a useful resource that may be used as a manual for resolving issues in connected subjects. The viability of applying neural networks to these kinds of issues is confirmed by this work, which also identifies areas for future development and application to more intricate systems. The accuracy of the network might be improved, and its application to a wider variety of differential equation problems could be investigated in future studies.

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